Grade 6 Math Circles
February 13 & 14 & 15, 2024
Knights and Knaves - Problem Set

1. What values can a variable in Boolean algebra take?

2. Pick one of the phrases below to complete the sentence: *In logic, the term "if and only if" between statements means that...*

   a. one of the two statements is true.

   b. both of the two statements are true.

   c. either the first statement is false and the second statement is true, or the first statement is true and the second statement is false.

   d. either both of the statements are true or both of the statements are false.

3. Let M be true and N be false. Assume that all variables in this question are boolean variables.

   (a) Let A represent the statement "Both M and N are true."
   What is the value of A?

   (b) Let B represent the statement "Either M is true or N is true."
   What is the value of B?

   (c) Let C represent the statement "Neither M nor N is true."
   What is the value of C?

   (d) Let D represent the statement "$M \iff N$."
   What is the value of D?

   (e) Let E represent the statement "$M \equiv N$."
   What is the value of E?

   (f) Let F represent the statement "M is not N."
   What is the value of F?

4. Let G represent the statement "Dakota has more berries than Quinn and Jayden is the same height as or taller than Elden." Suppose that G is false. What can we conclude? Avoid using the word *not* in your answer.

5. Let H represent the statement "Everyone in Ms. Greenspan's class saw the 2023 film *Barbie*." Suppose that H is false. What can we conclude? You may use the word *not* in your answer if you wish.

6. Let A, B, and C be Boolean variables. Complete the following truth table.

A	B	C	Exactly one of A and B are true	At least one of A, B, and C is true.
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

7. In this question, we will solve the following Knights and Knaves problem by constructing a truth table.

> I meet two inhabitants of an island that consists of only Knights and Knaves. I ask one of them, "Is either of you a Knight?" Based on that person's response, I know my answer. What are each of the two inhabitants—are they Knights or Knaves?

(a) Let A represent the statement "The person who I ask the question to is a Knight."
Let B represent the statement "The other person I meet who I do not ask the question to is a Knight."

Fill in the following truth table.

A	B	Either of the two people I meet is a Knight	The person I ask responds "Yes"
T	T		
T	F		
F	T		
F	F		

(b) Based on your truth table, what are each of the inhabitants? Explain your reasoning.

8. In this question, we will use a truth table to solve a Knights and Knave problem that involves three people. The problem is as follows.

> There are three people, Abby, Baloo, and Courtney, each of whom is either a Knight or a Knave. Abby and Baloo make the following statements:
>
> Abby: All of us are Knaves.
> Baloo: Exactly one of us is a Knight.
>
> What are Abby, Baloo, and Courtney?

(a) Let A represent the statement "Abby is a Knight."
Let B represent the statement "Baloo is a Knight."
Let C represent the statement "Courtney is a Knight."

Fill in the following truth table.

A	B	C	A, B, and C are all false	Exactly one of A, B, and C is true.
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

(b) Based on your truth table, what are each of Abby, Baloo, and Courtney? Explain your reasoning.

9. A barber of a certain small town shaved all inhabitants of the town who did not shave themselves, and never shaved any inhabitant who did shave themself. Explain why this situation is impossible. *Note: This is known as the Barber paradox.*

10. Imagine that you are trying to find an island called Treasure Island. You know that you are on an island of only Knights and Knaves. You meet two inhabitants who make the following

statements:

A: B is a Knight and this is Treasure Island.

B: A is a Knave and this is Treasure Island.

Is this Treasure Island? Justify your answer. You may use any method you wish.

11. Rama and Mysa are famous decorators of caskets. Whenever Rama decorates his caskets, he puts a true inscription on it, whereas Mysa puts only false inscriptions on his caskets. A jewel is in exactly one of three caskets with the inscriptions below, each decorated by either Rama or Mysa.

Casket 1: The Jewel is in here.
Casket 2: The Jewel is in here.
Casket 3: At least two of these caskets were decorated by Mysa

Determine the maker of each casket and choose the casket that contains the jewel. Justify your answer. You may use any method you wish.